Analytical Analysis: Shape and Mechanical Forces

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1.0 Introduction

The issue addressed by this analysis is the strength of the prosthetic forearm. This analysis involves calculating the forces on differently shaped forearms. This is important to the project because the arm needs to support the forces that are applied to it. Thus, the optimal shapes must be chosen to increase the durability of the prosthetic. This is done by treating the forearm as a cantilever beam. The forearm will be treated as a cantilever beam because it will be fixed to the patient's limb much like a cantilever is fixed in one location. From this analysis the necessary force to hold the prosthetic securely on the residual limb will be found. This is important because the attachment to the amputee must be strong and not fall off the user. The creation of Free Body Diagrams and Excel code to display the effects that different forearm design shapes will have on reaction forces, moments, and arm deflection at the joint. This will also factor in the location of forces. These forces can be distributed loads or point loads. In addition, different cross section shapes will be analyzed. The cross section affects the moment of inertia of beam. The change in shape will change the deflection and the bending stress is affected. This is important to the prosthetic, because the user should know the amount of force the arm can withstand without bending and breaking. So, this analysis will discover the optimal shape for a forearm that will withstand forces and the required reaction forces at the joint will be known. Thus, the proper shape and joint can be selected for the prosthetic.

2.0 Assumptions

For this analysis there are many assumptions made. The first and main assumption is that the forearm can be compared to a cantilever beam. The prosthetic arm will need to be secured to the residual limb. This will be done by fixing one end of the arm to the limb. This provides one fixed location to hold the weight of the arm and any other forces applied. Thus, it is comparable to a cantilever beam that is, by definition, fixed in one location.

It is also assumed that the weight forces of the arm are distributed loads and that the weight of the hand is a point load located at the end of the forearm. In addition, it is assumed that the arm is uniformly straight. All of these assumptions will cause the calculations to be simpler.

3.0 Physical Modeling

The Cantilever Beams are represented in the following figures. The first figure shows a physical image of a cantilever beam.

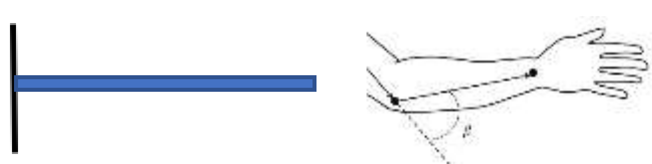

Figure 3.1: Cantilever Beam and Forearm Comparison [1]

This figure shows a cantilever beam alongside an arm to show the similarities of the beam to an arm. The cantilever is fixed at one end. Likewise the forearm is fixed at the elbow. Thus, the cantilever can be used to model the arm.

The cantilever and the arm must support many loads and load types. To simplify the calculations this will focus on the basic point and distributed loads.

The first type is a point load. This would include the weight of the hand or any objects that the user may hold in their hand. These loads are concentrated at one location and can be observed in the free body diagram below.

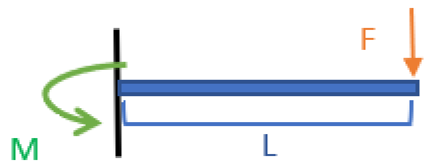

Figure 3.2: Free Body Diagram Cantilever Beam Point Load

This figure shows the point load on the beam of a length that will be determined by the user's anatomical dimensions. This free body diagram is used to find the shear and moment diagrams seen in the figures below.

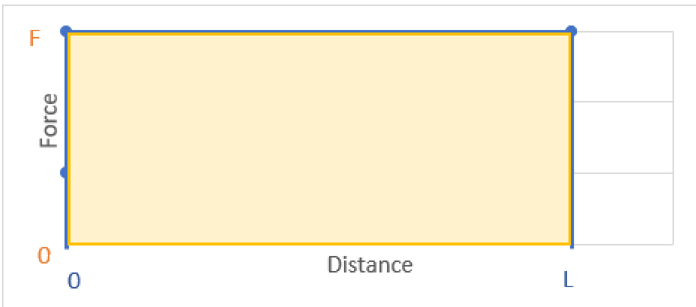

Figure 3.3: Shear Force Diagram Cantilever Beam Point Load

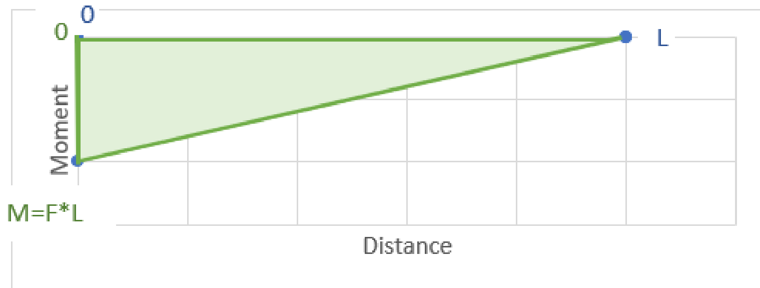

Figure 3.4: Bending Moment Diagram Cantilever Beam Point Load

The shear and moment diagrams show the reaction forces and torques that will be affecting the forearm at the joint. The shape of these diagrams will remain the same but change in magnitude based on the size of the force and the distance along the beam. These will be vital to find the bending stress of the forearm. Thus, it is important to know the force and moment.

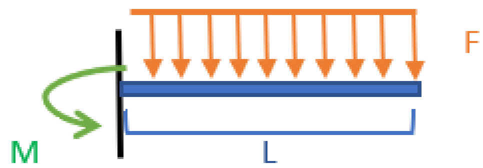

Figure 3.5: Free Body Diagram Cantilever Beam Distributed Load

This figure shows the distributed load on the beam of a length that will be determined by the user's anatomical dimensions. This free body diagram is used to find the shear and moment diagrams seen in the figures below.

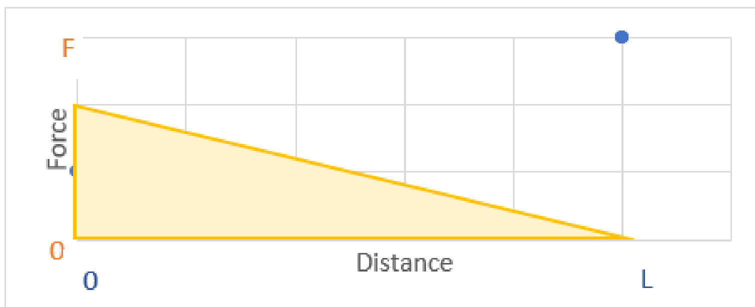

Figure 3.6: Shear Force Diagram Cantilever Beam Distributed Load

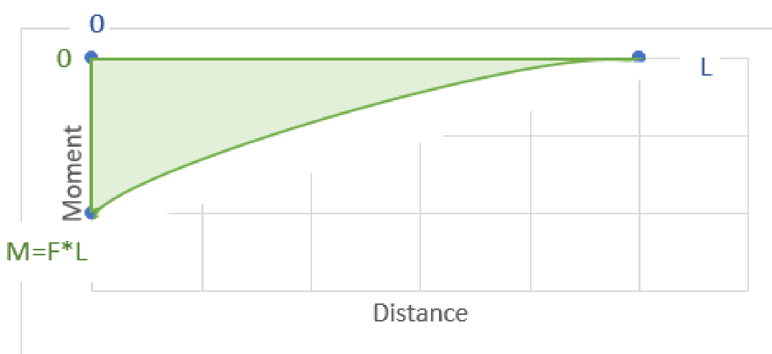

Figure 3.7: Bending Moment Diagram Cantilever Beam Distributed Load

The shear and moment diagrams show the reaction forces and torques that will be affecting the forearm at the joint. The shape of these diagrams will remain the same but change in magnitude based on the size of the force and the distance along the beam. It is important to know the force and moment. These will be vital to find the bending stress of the forearm.

The equations for finding the reaction forces and moments are commonly known formulas and are as follows [2].

The forces (F) are found based on mass (m) and acceleration (a). The masses can include the weight of objects and acceleration includes gravity or the movement of the arm to preform actions.

$$F = ma \tag{1}$$

The formula for moment (M) is the force (F) times the distance (L) along the beam.

$$M = FL \tag{2}$$

In order to find the reaction forces and moments the sum of forces and moments are taken.

$$\sum F = 0 \tag{3a}$$

$$\sum M = 0 \tag{3b}$$

The bending stress is a function of the moment (M), the moment of Inertia (I), and the vertical distance) [2].

$$\sigma = -\frac{MY}{I} \tag{4}$$

The deflection along the beam is also important to find those equations are different based on the type of load.

Load at free end:

$$\delta = \frac{Fl^3}{3EI} \tag{5}$$

The deflection in this case is affected by the modulus of elasticity (E), force, moment of inertia, and length (l) to the location of the load which is the end of the beam. This would include the weight of the hand and any objects held in the hand.

Load at any point:

$$\delta = \frac{FL^2}{3EI}(3l - L) \tag{6}$$

The deflection in this case is affected by the modulus of elasticity (E), force, moment of inertia, and length (l) to the location of the load, and the length of the beam (L). This would include the weight of any point loads not held in the hand.

Distributed load:

$$\delta = \frac{\omega l^4}{8EI} \tag{7}$$

The deflection in this case is affected by the modulus of elasticity (E), the distributed load ω , moment of inertia, and length (l) to the location of the load. This would include the weight of the forearm.

4.0 Diagrams for Moment of Inertia

In order to calculate the bending stress and the deflection of the arm, the moment of inertia must be found for each cross-sectional shape. Each of the shapes have unique moments of inertia as seen in the following figures and equations.

Cross-Sectional Shapes

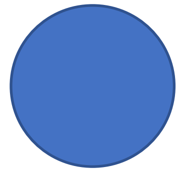

Figure 4.1: Cross-section Assuming Arm is Circular [2]

$$I = \frac{\pi D^4}{64} \tag{9}$$

The moment of inertia (I) depends on diameter (D).

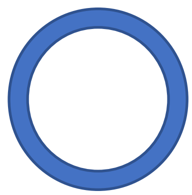

Figure 4.2: Cross-section Assuming Arm is Circular and Hollow [2]

$$I = \frac{\pi}{64}(D^4 - d^4) \tag{10}$$

The moment of inertia (I) depends on external diameter (D) and internal diameter (d).

Figure 4.3: Cross-section Assuming Arm is Semicircular and Hollow [2]

$$I = \frac{\pi}{8}(R^4 - r^4) \tag{12}$$

The moment of inertia (I) depends on external radius (r) and internal radius (r).

Figure 4.4: Cross-section Assuming Arm is Square and Hollow [2]

$$I = \frac{b^3 h}{12} \tag{11}$$

The moment of inertia (I) depends the length of the base (b) and the height (h).

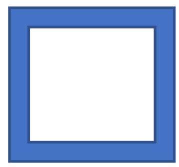

Figure 4.5: Cross-section Assuming Arm is Square and Hollow [2]

$$I = \frac{B^3 H}{12} - \frac{b^3 h}{12} \tag{12}$$

The moment of inertia (I) depends the length of the internal (b) and external (B) base and the internal (h) and external (H) height.

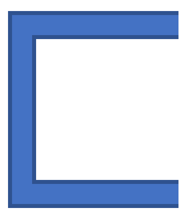

Figure 4.6: Cross-section Assuming Arm is "C" shaped and Hollow [3]

$$I = \frac{2sb^3ht^3}{3} - A(b-y)^2 \tag{12}$$

The moment of inertia (I) depends the length of the internal (b) and external (B) base, the internal (h) and external (H) height, and the base thickness (t) and height thickness (s).

5.0 Results

The results for the analysis were calculated in excel. These results varied based on dimensions, load sizes, and the Modulus of elasticity of the forearm. The inputs can be changing in the attached excel sheet to fit the needs of the individual user. The results show that the shape with the larges moment of inertia resulted in the smallest deflection and the smallest bending stress. this shape is the hollow semicircle. Therefore, the team should consider this shape for the design of the forearm.

Reference

[1] http://www.scielo.br/scielo.php?script=sci_arttext&pid=S1980-65742014000100033

[2] Shigley's Mechanical Engineering Design ninth edition

[3] L. L. C. E. Edge, “Moment of Inertia Equation, Section Modulus Equation, Radii of Gyration Equations Channel Sections,” Engineers Edge. [Online]. Available: https://www.engineersedge.com/material_science/moment-inertia-gyration-5.htm. [Accessed: 10-Nov-2018].